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# Method of Verification of Hypothesis about Mean Value on a Basis of Expansion in a Space with Generating Element

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**Abstract**—In this paper it is proposed an original method for verification of statistical hypotheses about mean values of random quantities. This method is based on Kunchenko stochastic polynomials tool and probabilistic description on a basis of higher order statistics (moments and/or cumulants). There are represented analytical expressions allowing to optimize decision rules using certain qualitive criterion and calculate decision-making error. It is shown polynomial decision rule in case of polynomial power $S = 1$ corresponds to classic linear decision rule which is used for comparative analysis. By means of multiple statistical experiments (Monte–Carlo method) obtained results of Neumann–Pierson criterion show proposed polynomial decision rules are characterized by increased accuracy (decrease of the 2nd genus errors probability) in compare to linear processing. The method efficiency increases with increase of stochastic polynomial order increase of degree of random quantities distribution difference from Gaussian probabilities distribution law.

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## 1. INTRODUCTION

It is known parametric approach to decision of the problem of statistic hypothesis verification allowing to obtain optimal solution is based on likelihood functional shaping which is calculated using empiric data. But practical implementation of such approach is accompanied by amount of information and computational problems. Moreover parametric methods can be characterized by essential complexity and inconvenience complicating their algorithmic realization and analysis of obtained solutions efficiency.

It leads to popularization of non-parametric statistics methods whose main advantage is their computational simplicity and absence of binding to specific distribution type. But “not accounting” of probabilistic properties of statistical data results in decrease of power of non-parametric criteria in compare to optimal parametric criteria.

One of compromise approach to statistic problems solution is building of probabilistic models on a basis of description of higher order statistics (moments or cumulants). Such description is incomplete, as a result there is only asymptotic possibility (with increase of parameters amount) of obtain of optimal methods of statistical processing on this basis. But its application is essentially more efficient from viewpoint of realization since it decreases essentially the requirements to a priori information and simplifies algorithmic realization of statistical processing methods.

The result is wide application of description tool of higher order statistics used for various problems solutions, for example, for detection and estimation of signals parameters [1–3], recognition and identification [4, 5], regression analysis [6–8], probabilistic and technical diagnosis [9–11], etc.

In this paper it is considered the problem of verification of simple hypothesis about mean value of random quantities. This problem represents one of the most typical examples of application of statistical hypothesis verification theory and it occurs in many fundamental papers of mathematical statistics [12, 13]. It can be explained by relative simplicity of such problems and great amount of application fields where this problem solution is required. In particular, such problems solution is key element for development of an algorithm for signals detection on a background of noise in statistical radio engineering [14], in communication field it is necessary for synthesis of procedures for recognition in detectors and demodulators of digital communication systems [15].

Examples of development of decision rules on a basis of functions of higher order sample statistic are experimental data approximation with Johnson’s [16] and Pierson’s [17] distributions for models with continuous time or Ord's distribution [18] for models with discrete time, and also D'Agostino's criterion [19] and Jarque–Bera test [20] used for verification of hypothesis about normality.

In many cases this problem solution is based on shaping of linear statistics in form of sample mean that is practically is estimation of this parameters obtained with moments method. It is known that linear methods of statistical processing are optimal only in case of normal (Gaussian) data distribution. In particular, in papers [21, 22] there are represented theoretical and experimental results of research of efficiency of estimations of measured parameter obtained on a basis of application of polynomial maximization method [23], which is an evidence of decrease of variance of non-linear estimations in case of non-Gaussian behavior of measurements error.

In this case for development of decision rules there are used Kunchenko's stochastic functional polynomials [24]. Moreover, there are applied optimization principles which are different from used in polynomial maximization methods which are based on the other properties of stochastic polynomials. Mathematic basis of these researches is tool of polynomial approximation in a space with generating element [25].

## 2. PURPOSE OF THE PAPER

The purpose of the paper is development and research of efficiency of decision-making method basing on tool of power stochastic polynomials and expansion in a space with generating element for verification of statistical hypothesis about numerical value of mean of random quantities with probabilities distribution which is different from Gaussian law.

## 3. PROBLEM STATEMENT

Let $\vec{X} = \{x_1, x_2, \ldots, x_N\}$ is a vector containing statistically independent identically distributed sample values of random quantity $\xi = a + \xi_0$ where $\xi_0$ is centered random quantity whose probabilistic behavior is defined by finite central moments $\mu_i$ or cumulants $\chi_i$, $i = \overline{1, 2S}$. Main hypothesis about random value mean $H_0$ is considered as situation where random value mean $\xi$ is equal to some value $a = a_0$. The other mean values ($a \neq a_0$) are considered as alternative hypothesis $H_1$.

It is necessary using tool of random quantities expansion in a space with generating element and power transformations to synthesize and analyze of decision rules in form of stochastic polynomials of $S$th order allowing to verify or disprove the realization of $H_0$ hypothesis.

## 4. RESEARCHES RESULTS

### *4.1. Linear Statistics*

According to classical linear approach to stated problem solution decision rule of the hypothesis verification uses following statistics

$$f_1(\vec{X}) = \frac{1}{N} \sum_{n=1}^{N} x_n - a, \tag{1}$$

which can be treated as a difference between sample $\hat{\alpha}_1$ and theoretical $\alpha_1 = a$ moments of the first order of the quantity $\xi$.

It is known statistics (1) is random value which according to central limit theorem is asymptotically distributed with Gaussian law independently on $\xi$-distribution. It is shown from paper's aim that in case of realization of hypothesis $H_0$ linear statistics $f_1(\vec{X})$ has zero mathematical expectation $E_1^{(0)} = 0$ and variance in form of

$$D_1^{(0)} = \sigma_\xi^2 / N. \tag{2}$$

Thus, accuracy of decision making is under the influence of two factors: volume of sample values $N$ and variance $\sigma_\xi^2$ of the random quantity $\xi$ whose value is equivalent to a value of the second order cumulant $\chi_2$ or the second central moment $\mu_2$ of the random quantity $\xi$.

### *4.2. Quadratic Statistics*

Analogously to the principle of statistics (1) shaping we can write quadratic statistics in following form:

$$f_2(\vec{X}) = \frac{1}{N}\sum_{n=1}^{N}(x_n)^2 - \alpha_2, \tag{3}$$

where $\alpha_2 = \chi_2 + a^2$ is initial moment of the second order of random quantity $\xi$.

It is obvious that quadratic statistics (3) can be also used for development of decision rule of verification of the hypothesis about mean of random quantity $\xi$ in case of known $\chi_2$.

In this case there is a question which statistics (1) or (3) is better. What is better error probability minimization or decision rule generation on a basis of combination of these statistics optimal from definite criterion?

The answer of this question can be given on a basis of analysis of main property of stochastic polynomials (Kunchenko's properties) lying in ability to reduce random quantity variance [24]. This property is generated by specificity of original method of polynomial approximation to an object (vector, determined function, random value or process) in a space with generating element [25]. Such expansion basis in general case is non-orthogonal and it consists of non-linear transformation definitely matched to generating object.

If we use power function as a basis than for random quantity $\xi$ such expansion can be represented in following form:

$$\xi = \alpha_1 + \sum_{i=2}^{\infty} k_i[\xi^i - \alpha_i], \tag{4}$$

where (4) is treated as equality in mean square.

In case of restriction of series member amount expansion inaccuracy appears:

$$\xi_S = \xi - \alpha_1 - \sum_{i=2}^{S} k_i[\xi^i - \alpha_i]. \tag{5}$$

This inaccuracy is such random quantity which in case of obtain of coefficients $k_i,\ i = \overline{2,S}$ from algebraic linear equations system in following form:

$$\sum_{i=2}^{S} k_i F_{i,j} = F_{1,j},$$

$$F_{i,j} = \alpha_{i+j} - \alpha_i\alpha_j, \quad j = \overline{2,S} \tag{6}$$

has the variance multiplied by factor of

$$\delta_S = 1 - \frac{\sum_{i=2}^{S} k_i F_{1,i}}{\chi_2}, \tag{7}$$

from the interval (0;1] is less than variance of generating random value $\xi$.

Therefore, in case of polynomial degree $S = 2$ we can shape the statistics

$$g_2(\vec{X}) = \frac{1}{N}\sum_{n=1}^{N}\left\{x_n - a - k_2\left[(x_n)^2 - (\chi_2 + a)\right]\right\}, \tag{8}$$

which is sample mean (linear estimation of mathematical expectation) of random quantity of following form:

$$\xi_2 = \xi - \alpha_1 - k_2(\xi^2 - \alpha_2).$$

Coefficient $k_2$ which are optimal from viewpoint of minimization of statistics (8) variance is obtained from the system (6) as follows:

$$k_2 = \frac{1}{\sqrt{\chi_2}}\frac{2\sqrt{q} + \gamma_3}{4q + 4\sqrt{q}\gamma_3 + \gamma_4 + 2}, \tag{9}$$

where $q = a^2/\chi_2$, and $\gamma_3 = \chi_3/\sqrt{\chi_2^3} = \mu_3/\sqrt{\mu_2^3}$, and $\gamma_4 = \chi_4/\chi_2^2 = \mu_4/\mu_2^2 - 3$ are cumulant coefficients of asymmetry and excess of random quantity $\xi$, correspondingly, and they characterize its non-gaussianity in case of their values are non-zero.

The value of coefficient of statistics (8) variance decrease calculated for $S = 2$ according to (7) in case of realization of expected hypothesis $H_0$ is defined by expression:

$$\delta_2 = 1 - \frac{(2\sqrt{q} + \gamma_3)^2}{4q + 4\sqrt{q}\gamma_3 + \gamma_4 + 2}. \tag{10}$$

We should note there is possible another alternative variant of decision rule building based on application of statistics in form of

$$t_2(\vec{X}) = \frac{1}{N}\sum_{n=1}^{N}\left\{x_n - a - h_2\left[(x_n - a)^2 - \chi_2\right]\right\}, \tag{11}$$

which is based on calculation of the difference between sample and theoretic central moments.

Optimal coefficient $h_2$ minimizing the variance of statistics (11) is simpler in compare to (9):

$$h_2 = \frac{1}{\sqrt{\chi_2}}\frac{\gamma_3}{\gamma_4 + 2},$$

and coefficient of decrease of statistics (11) variance depends on cumulant coefficients of asymmetry and excess only:

$$\delta_2 = 1 - \frac{(\gamma_3)^2}{\gamma_4 + 2}. \tag{12}$$

We note decrease of the variance of statistics (8) and (11) by corresponding values (10) and (12) is achieved due to additional a priori information in form of values of parameters (moments or cumulants) up to the 4th order of random quantity $\xi$.

### *4.3. Polynomial Statistics*

Taking into account the results represented in previous section we can write in general form the expressions for polynomial statistics of two types based on initial and central moments correspondingly:

$$g_S(\vec{X}) = \frac{1}{N}\sum_{n=1}^{N}\sum_{i=1}^{S} k_i\Big[(x_n)^i - \alpha_i\Big], \tag{13}$$

$$t_S(\vec{X}) = \frac{1}{N}\sum_{n=1}^{N}\sum_{i=1}^{S} h_i\Big[(x_n - a)^i - \mu_i\Big], \tag{14}$$

where there are introduced additional unit coefficients $k_1 = h_1 = 1$ for generalization, and the values of the other optimal coefficients are obtained from (6) and inverted (multiplied by "–1").

As it is mentioned above statistics in form of (13) and (14) represent sample mean of random quantity which is shaped as difference between generating value and its matched representation in form of stochastic polynomial. Form the other hand such statistics can be treated as weighted by optimal coefficients average values of difference between theoretic and experimental values of mathematical expectation of stochastic polynomials basis functions. It is shown [24] that mean values of stochastic polynomials in form of (13) and (14) are asymptotically distributed with Gaussian law which is corollary fact of central limit theorem proved in Kunchenko's form.

As it is known parameters of Gaussian distribution are mathematical expectation *E* and variance *D*. The values of these parameters of corresponding statistics essentially depend on realization of expected hypothesis $H_0$ whose optimal expansion coefficients are obtained or its alternative $H_1$.

It can be easily shown in optimal case (in case of realization of hypothesis $H_0$) mathematical expectation of statistics (13) and (14) is equal to zero ($E_S^{(0)} = 0$) and variance of statistics (13) can be defined according general relation:

$$D_S^{(0)} = \frac{1}{N}\left[F_{1,1} - \sum_{i=2}^{S} k_i F_{1,i}\right] = \frac{\sigma_\xi^2}{N}\left[1 - \frac{J_S}{\sigma_\xi^2}\right], \tag{15}$$

where the value

$$J_S = \sum_{i=1}^{S} k_i F_{1,i}$$

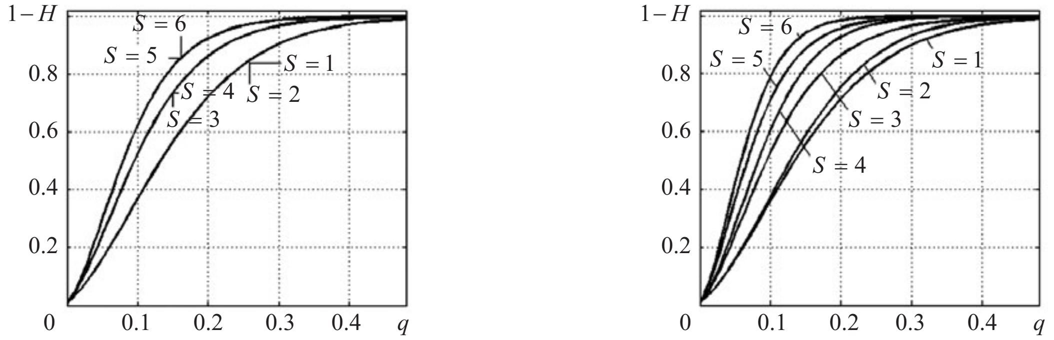
is called inforkune, whose value characterizes degree of the variance reduction regarding the variance of generating random quantity [24].

Taking into account fact that increase of series members inforkune value increases monotonically and it tends to value of generating random quantity variance ($\lim_{S\to\infty} J_S = \sigma_\xi^2$) then there is additional way for decrease of the variances of statistics (13) and (14) even in case of fixed volume of sample *N*.

In case of alternating situation in case of $H_1$ realization where mean value of random quantity $\xi$ is different from expected value ($\alpha_1^{(1)} \neq a$) statistics (13) parameters can be defined using following formulas:

$$E_S^{(1)} = \sum_{i=1}^{S} k_i^{(0)}[\alpha_i^{(1)} - \alpha_i^{(0)}], \tag{16}$$

$$D_S^{(1)} = \frac{1}{N}\left[P_{1,1}^{(10)} - 2\sum_{i=2}^{S} k_i^{(0)} P_i^{(10)} + \sum_{i=2}^{S}\sum_{j=2}^{S} k_i^{(0)} k_j^{(0)} P_{i,j}^{(10)} - (E_S^{(1)})^2\right], \tag{17}$$



(a) (b)

**Fig. 1.** Dependences of detection characteristics on polynomial power: for excess (symmetrically-distributed) random quantities with parameters $\gamma_3 = 0, \gamma_4 = 5$ (a); for asymmetrically-excess random quantities with parameters $\gamma_3 = 1, \gamma_4 = 5$ (b).

where $P^{(10)}_{i,j} = \alpha^{(1)}_{i+j} - \alpha^{(1)}_{i}\alpha^{(0)}_{j} - \alpha^{(0)}_{i}\alpha^{(1)}_{j} + \alpha^{(0)}_{i}\alpha^{(0)}_{j}$.

Indexes in parentheses in (16) and (17) define type of the hypothesis where following parameters must be calculated: moments, correlates and optimal coefficients.

We should note represented expressions (15)–(17) define parameters of statistics (13) which is based on initial moments $\alpha_i$ of random quantity $\xi$. But these relations can be applied also for calculation of corresponding statistics (14) parameters if we substitute initial moments $\alpha_i$ by central ones $\mu_i$ in all calculation expressions.

### *4.4. Generation of Decision Rules and Their Characteristics*

Uniformity (asymptotic normality) of distributions of classic linear statistics (1) and polynomial statistics (13) and (14) allows to apply similar principles of decision rules generation. It is obvious direct application of such statistics which in case of expected hypothesis have symmetrical distribution with respect to zero requires application of double-sided criteria for decision making that is inconvenient in some cases.

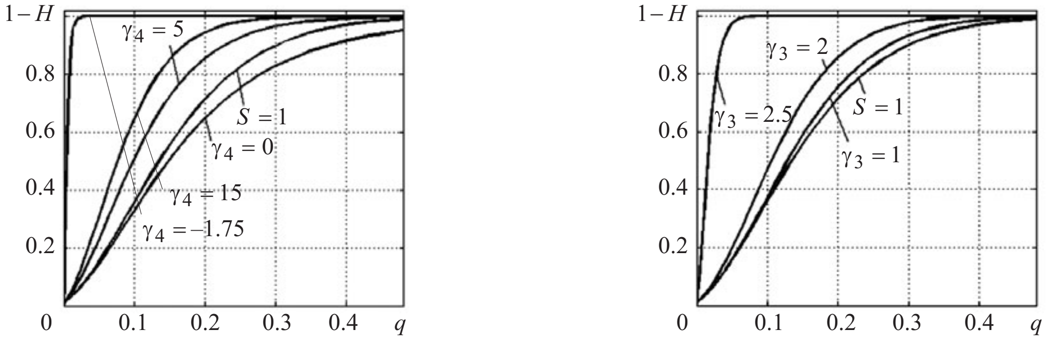
For transition to one-sided criterion, for example, right-sided criterion we can modify these statistics taking their absolute value square. Both distributions are invariant from viewpoint of decision-making accuracy. In the first case resulting decision rule in case of some statistics $f(\vec{X})$ can be represented in following form:

$$|f(\vec{X})| \underset{H_1}{\overset{H_0}{\lessgtr}} C.$$

Taking into account presence of all calculation relations which allow to define parameters of distribution of decision-making function in case of realization of the hypothesis and alternative we can realize analytical calculation of probabilities of erroneous solutions of the 1st and 2nd genus and also optimize the selection of the values of the decision threshold $C$ according to definite qualitive criterion (maximal likelihood, average risk, Neumann–Pierson, etc.).

We represent below graphic results of the researches obtained by means of statistical modeling for the simplest by important from viewpoint of application where mathematical expectation in case of hypothesis $H_0$ is equal to zero. From position of statistical radio engineering such situation can be generated by typical problem of the signals detection when realization of hypothesis $H_0$ can be treated as a presence noise only in channel and its alternating realization $H_1$ is treated as presence of a signal with unknown amplitude.

In case of such problem statement the most spread is application of Neumann–Pierson criterion, according to which the threshold $C$ is calculated in case of fixation of false alarm probability $F$, and efficiency of algorithms of hypotheses verification is defined by correct detection characteristics, i.e. by dependences of the value $1 - H$ on power signal-to-noise ratio $q = a^2 / \chi_2$. here $H$ is probability of the signal miss.



(a) (b)

**Fig. 2.** Dependences of detection characteristics on non-gaussianity degree: for excess (symmetrically-distributed) random quantities in case of polynomial power $S = 3$ (a); for asymmetrically-excess random quantities ($\gamma_4 = 5$) in case of polynomial power $S = 2$ (b).

In Figs. 1, 2 there are represented the dependences of detection characteristics built in case of the sample volume $N = 50$ and fixed values of false alarm probability $F = 10^{-2}$. As a model of noise component we use random quantity with bigaussian distribution whose application is characterized by convenience for analytical calculation together with high functionality for the width of fixation of the values of cumulant coefficients of asymmetry and excess [26].

## 5. ANALYSIS RESULTS

Analysis of obtained results allows to conclude following main moments:

– polynomial decision rules in case of polynomial power $S = 1$ matches classic linear decision rule;

– in case of non-Gaussian behavior of random quantities increase of polynomial power results in detection accuracy increase (decrease of error probability);

– for random quantities with symmetric distribution the accuracy is modified only by increase of polynomial power from pair order to unpaired one that is explained by zero pair coefficients of polynomials for this case;

– increase of degree of non-gaussianality of random quantities results in increase of decisions accuracy that can be essential in case of proximity of absolute values of cumulant coefficients of higher orders to limit of their accepted values;

– in case of close to Gaussian behavior of random quantities decision-making accuracy in case of definite degrees of stochastic polynomials can be less than accuracy of linear decision rule.

## 6. CONCLUSIONS

It is proposed principally new approach to development of the algorithms for verification of statistical hypotheses about numerical value of random quantity mean based on decision function represents stochastic power polynomial whose coefficients are obtained from condition of its variance minimization. It gives the possibility to decrease critical area of hypothesis acceptation even in case of fixed volume of sample values. Obtained effect is achieved due to taking into account the values of higher order statistics of random quantity (cumulant coefficients) in decision functions.

At whole obtained polynomial decision rules are characterized by increased accuracy in compare to linear processing increasing with increase of polynomial order and increase of degree of the difference of random quantities distributions from Gaussian law. This gain is achieved due to calculations complication and condition of additional information about expected hypothesis in form of moments sequence which are necessary for calculation of optimal parameters of shaped polynomial statistics.

Since proposed approach does not use information about statistical data distribution law but nevertheless its probabilistic character is taken into account then resulting method of the hypothesis verification can be referred to intermediate class of statistical methods which are often called semi-parametric.

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