

<https://doi.org/10.15407/scine22.04.090>

PASICHNYK, T. O. (<https://orcid.org/0000-0002-7284-7345>)

University of Customs and Finance,
2/4, Vernadskoho St., Dnipro, 49000, Ukraine,
+380 56 745 5596, umsf.ua@gmail.com

BALANCED GROWTH IN THE SOLOW–SWAN MODEL: THE ROLE OF HARROD-NEUTRAL TECHNICAL PROGRESS

Introduction. *The Solow–Swan model with a neoclassical production function, constant returns to scale, and diminishing marginal productivity of production factors has remained a standard framework for macroeconomic growth analysis.*

Problem Statement. *Aligning the model with long-run macroeconomic data series has required clarification of the nature of technical change and the measurement of factor contributions. This issue has remained important both theoretically and practically, since debates concerning the “productivity slow-down” and the role of intangible assets have significantly affected the assessment of growth sources and the formulation of economic policy.*

Purpose. *To develop and empirically test a methodology for constructing growth paths within the Solow–Swan model with Harrod-neutral technical progress: (a) a discrete version with time-varying parameters; (b) a continuous version with averaged parameters; and growth accounting based on alternative definitions of production factors in order to isolate the contribution of technical progress.*

Materials and Methods. *The primary source of macroeconomic time series has been the Penn World Table (10.01). The analysis has been conducted in both discrete and continuous settings. Technical progress has been estimated using the Solow residual approach, while sensitivity analysis has been performed for alternative measures of labor, capital, and factor shares.*

Results. *The Cobb–Douglas specification with Harrod-neutral technical progress has demonstrated a good fit for mature economies. At the same time, for transition economies characterized by frequent structural breaks, the absence of time calibration has biased steady-state estimates and reduced the model’s empirical fit.*

Citation: Pasichnyk, T. O. (2026). Balanced Growth in the Solow–Swan Model: the Role of Harrod-Neutral Technical Progress. *Sci. innov.*, 22(4), 90—107. <https://doi.org/10.15407/scine22.04.090>

© Publisher PH “Akadempriodyka” of the NAS of Ukraine, 2026. This is an open access article under the CC BY-NC-ND license (<https://creativecommons.org/licenses/by-nc-nd/4.0/>)

Conclusions. Total factor productivity (TFP) should be interpreted with caution, since it incorporates not only technological change, but also organizational transformation, allocative efficiency, capacity utilization, and measurement errors. The inclusion of intangible assets in the capital stock has partially shifted the explanation of growth from the residual component toward measurable production factors. At the same time, a limitation of models with exogenous technical progress and perfect competition has remained their inability to capture innovation as purposeful economic activity. Addressing this limitation requires the application of endogenous growth models that account for innovation rents and incentive mechanisms.

Keywords: balanced growth, growth accounting, intangible capital, factor shares, technical progress, innovative development.

Research on economic growth generally distinguishes between two fundamental paradigms: models with exogenous technical progress and models with endogenous technical progress; alongside them, semi-endogenous models have also been developed. Historically, models with an exogenously determined technological trajectory emerged first. In exogenous growth models, technology A (or its growth rate g), the population growth rate n , and the depreciation rate δ are treated as exogenous variables. The behavior of economic agents (savings, investment, and consumption) may be specified exogenously, as in the Robert Solow–Trevor Swan model, where the savings rate s is fixed either as a constant or through external policy decisions. Alternatively, it may be determined endogenously through an intertemporal optimization problem, as in the Frank Ramsey–David Cass–Tjalling Koopmans model, in which economic trajectories follow from optimality conditions and the savings rate emerges as the outcome of agents' decisions.

The attractiveness of exogenous models lies in their simplicity, the transparency of their analytical results, and their convenience for empirical research. Such models can be directly calibrated using statistical series of total factor productivity (hereafter, TFP) or specified as stochastic processes for comparative analysis across countries and time periods. Growth accounting serves as a fundamental tool for decomposing the contributions of capital, labor, and TFP, as well as for constructing conditional forecasts under a given technological trajectory.

The exogeneity of technical progress is a defining feature of canonical neoclassical growth models. The issue is not whether innovation at the firm le-

vel is rational or irrational; rather, it is a methodological assumption whereby long-run technological change is treated as an external process, represented either as a deterministic trend or as an exogenous shock. This approach makes it possible to analyze the implications of technological change for equilibrium without modeling the micro-level mechanisms underlying innovation. At the same time, neoclassical models assume that technological shifts alter equilibrium factor prices and influence accumulation dynamics; therefore, technical progress is modeled as an exogenous process A_t that multiplicatively shifts the production function.

Nevertheless, interwar economic literature had already begun to move toward the endogenization of technology. In particular, Allyn Young (1928) emphasized the role of product diversification and increasing returns generated through Marshallian externalities. Within this framework, innovations introduced by individual firms increase the aggregate productivity of the entire sector [1].

Early neoclassical approaches, following the tradition of classical political economy, treated capital and labor as the principal production factors for modeling long-run growth. The key role was assigned to capital accumulation, whereas technical progress was assumed to be exogenous. For some time, the Roy Harrod–Evsey Domar [2, 3] model remained influential, emphasizing potential growth dysfunctions: the economy may continue to expand while unemployment rises when the actual growth rate deviates from the “natural” or “warranted” rate. However, the principal working framework of macroeconomic analysis eventually became the Robert Solow–Trevor Swan [4, 5] mo-

del with a neoclassical production function characterized by constant returns to scale and diminishing marginal productivity of production factors (hereafter referred to as the Solow model).

In the contemporary literature on the Solow model, two aspects are of primary interest: the role of technical progress in economic growth and the estimation of TFP. The necessity of Harrod-neutral technological change for balanced growth is assessed differently across studies. In [6], it is stated that this condition is not mandatory and that interpreting the parameter A in the production function $Y = AF(K, L)$ as representing “neutral” technical progress constitutes a methodological error. At the same time, the study emphasizes that growth accounting remains valid if A is interpreted as residual TFP and technological change is modeled jointly with possible parameter variation.

In [7], Solow-neutral technical progress was reconciled with long-run equilibrium by defining the “natural” growth rate as the growth rate of effective capital, demonstrating its compatibility with constant returns to scale, and deriving the equation of a concave production possibility frontier that allows for positive growth and its outward shift under appropriate conditions.

In [8], it was shown that in the Solow model with CES technology and labor-saving technical progress, a balanced growth path exists only under a sufficiently high savings rate; otherwise, a “polar” regime emerges, characterized by a constrained growth rate and an increasing capital share in output. In [9], localized technical progress was examined within the Solow framework: an increase in the investment rate raises the long-run growth rate along the balanced path, while locally biased technological change gradually reduces labor’s share in output and increases the capital share. In [10], it was demonstrated that when investment simultaneously increases both the quantity and efficiency of production factors, a steady state requires Hicks-neutral rather than Harrod-neutral technological change.

The literature on TFP estimation encompasses both methodological and empirical contributions. Analysts at the World Bank [11] identified five de-

terminants of TFP — innovation, education, market efficiency, infrastructure, and institutions — constructed annual indices for more than 100 countries over the period 1985–2015, and demonstrated a positive relationship between TFP growth and the aggregated five-factor index. In [12], TFP in manufacturing was estimated using the AMG method on data from 63 countries covering 1980–2020. In [13], four approaches to TFP measurement (OP, LP, Wooldridge, and ACF) were compared using cross-country data, with preference given to the approach developed by Jeffrey Wooldridge [14] due to its lower variance and more stable, statistically significant estimates. Study [15] proposed a growth-accounting framework with time-varying factor shares for 101 countries over 1950–2015 and subsequently explained the contributions through panel regressions accounting for heterogeneity, cross-sectional dependence, and autocorrelation. Research [16] demonstrated a “two-dimensional” measurement of productivity, distinguishing between labor-saving and capital-saving technological change by identifying them through fluctuations in factor shares.

Despite the analytical clarity and widespread application of the Solow model, reconciling theory with long-run time series data requires a more precise specification of the nature of technical progress and the methods used to measure factor contributions. This issue carries both theoretical and practical significance: debates concerning the “productivity slowdown” and the role of intangible assets directly affect the assessment of growth sources and, consequently, economic policy. Under conditions of digitalization, which substantially increase the importance of intangible capital and blur the distinction between investment and current expenditures, it becomes critically important to define clearly what should be treated as “capital” and, accordingly, to measure factor contributions correctly.

The purpose of the article is to develop and empirically test a methodology for constructing growth trajectories in the Robert Solow–Trevor Swan model with Harrod-neutral technical progress: (a) in

a discrete-time specification with time-varying parameters; (b) in a continuous-time specification with average parameter values; and (c) through growth accounting (calculation of the “Solow residual”) using several alternative approaches to factor definition in order to isolate the contribution of technical progress to overall economic growth.

ANALYTICAL SPECIFICATION OF THE SOLOW MODEL WITH HARROD-NEUTRAL TECHNOLOGY

Consider the neoclassical Solow growth model with a constant saving rate and exogenous technological progress. The economy is closed and produces a single final good; neither government nor foreign sectors are included. Full employment is assumed, and labor supply L_t is perfectly inelastic and independent of the wage rate, w_t . Unlike the Ramsey–Cass–Koopmans model, intertemporal consumption optimization is not specified: household preferences and budget constraints are left unspecified, while the saving rate, $s \in (0, 1)$, is assumed to be constant and exogenous. Consequently, in a closed economy, $S_t = I_t = sY_t$.

Households own the capital stock and rent it to firms at the competitive rental rate, R_t . Under perfect competition, both the wage rate and the rental rate of capital w_t and R_t are equal to the marginal products of the respective factors of production. This exogenously determined saving rule, rather than one derived from utility maximization, simplifies the characterization of market equilibrium and allows the analysis to focus on the dynamics of capital accumulation and the role of exogenous technological progress, A_t .

We assume perfect competition and unrestricted access to technology for all firms. Under constant returns to scale, the aggregate economy can be represented as if production were carried out by a single representative firm with the production function

$$Y_t = F(K_t, L_t, A_t), \quad (1)$$

where Y_t denotes final yield/output, K_t is the stock of physical capital, L_t is employment, and A_t is the level of technology at time t .

The production function F is assumed to be continuous and twice continuously differentiable with respect to K and L . Both factors exhibit positive but diminishing marginal products ($F_K > 0$, $F_{KK} < 0$, $F_L > 0$, $F_{LL} < 0$); the function is linearly homogeneous in capital and labor (K and L) for a given level of technology A , implying constant returns to scale, and is strictly concave. In addition, the Inada conditions are assumed to hold:

$$\lim_{K \rightarrow 0} F_K(K, L, A) = \infty, \quad \lim_{K \rightarrow \infty} F_K(K, L, A) = 0$$

$$(\forall L > 0, A > 0),$$

$$\lim_{L \rightarrow 0} F_L(K, L, A) = \infty, \quad \lim_{L \rightarrow \infty} F_L(K, L, A) = 0$$

$$(\forall K > 0, A > 0).$$

Under these assumptions, sustained long-run growth in output per worker is possible only in the presence of technological progress. In its absence, the economy converges, after a transitional adjustment period, to a steady-state level in which output per worker remains constant. During the transition, growth may occur even without technological progress through increases in capital intensity; however, such growth is temporary and gradually dissipates as the economy approaches its steady state.

For the subsequent analysis, the general production function F is specified as a Cobb–Douglas production function, which satisfies the properties outlined above:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad (2)$$

where α denotes the output elasticity with respect to capital (and, correspondingly, $1 - \alpha$ with respect to labor). This specification simultaneously captures diminishing marginal returns to individual factors and constant returns to scale.

It is important to interpret the parameter A_t correctly. In the narrow sense, it represents a Hicks-neutral multiplier that increases the productivity of both factors and shifts the production function upward. In a broader sense, it corresponds to TFP, i.e., an indicator of the “efficiency” of the economy that encompasses not only engineering-level technology but also production organization, insti-

tutional quality, allocative efficiency, capacity utilization, and even measurement errors. Therefore, it is more appropriate to refer to it as TFP or efficiency, while recognizing its inherently composite nature.

Physical capital is accumulated through investment; its continuous-time dynamics are given by the formula:

$$\dot{K}_t = I_t - \delta K_t, \quad (3)$$

where $\dot{K}_t$ denotes the change in the capital stock (net investment); I_t represents gross investment; and $\delta \in (0, 1)$ is the depreciation rate. In a closed economy without government intervention and with a constant saving rate s , we have $I_t = sY_t$ and consequently $C_t = (1 - s)Y_t$.

The key variable is capital intensity, $k_t \equiv K_t / L_t$. Let output per worker $y_t \equiv Y_t / L_t$ be denoted by y and consumption per worker by $c_t \equiv C_t / L_t$. Under constant returns to scale, we obtain:

$$y_t = F\left(\frac{K_t}{L_t}, 1, A_t\right) \equiv f(k_t, A_t) \quad (4)$$

or, for concreteness, under the Cobb–Douglas specification in (2), we obtain:

$$y_t = A_t k_t^\alpha, \quad 0 < \alpha < 1. \quad (5)$$

In the context of growth models, we are concerned with three related notions of equilibrium: (1) competitive market equilibrium in each period (Walrasian market clearing at each time t , where factor prices equal marginal products; a full definition of such equilibrium is provided in [17]); (2) the steady state (long-run equilibrium); and (3) the balanced growth path, consistent with Kaldor’s “stylized facts” [18].

The steady state in the Solow model is an equilibrium path along which capital intensity remains constant over time, i.e., $k_t = k^*$ for all t (equivalently, $\dot{k} = 0$). Intuitively, it represents a “rest point” of the dynamical system. In reality, most economies are away from the steady state at any given time, while simultaneously achieving competitive equilibrium in each period. Therefore, not only the terminal position matters, but also the transitional dynamics — the equilibrium adjustment path through which k_t converges to k^* (“convergence”).

If technology does not grow ($A = \text{const}$), then under diminishing marginal productivity of capital, output, consumption, and capital per worker become constant in the long run: per capita growth disappears, and the economy converges to a steady state. This is consistent with classical intuition: capital accumulation without technological progress is not sufficient to sustain long-run growth in labor productivity.

However, this restriction of the model is at odds with empirical evidence: in most developed countries, average GDP per capita growth rates have been positive over the past two centuries [19]. In the absence of technological progress and under diminishing marginal returns to factors, simple accumulation of capital per worker cannot sustain long-run growth in per capita income; the economy converges to a steady state. This conclusion echoes the ideas of T. Malthus and D. Ricardo and similarly relies on the assumption of diminishing returns to capital.

Let us turn to the third type of equilibrium — balanced growth equilibrium, which is grounded in Kaldor’s “stylized facts” [18]. Based on empirical observations, N. Kaldor established that during economic growth, despite rising output per capita, several ratios remain relatively stable: the capital–output ratio, the real interest rate, and the income shares of capital and labor. These regularities form the basis of the balanced growth concept, under which the aforementioned variables remain constant over time. Although this is only an approximation of reality, economists often prefer balanced growth models because they are analytically more tractable. In particular, empirical studies frequently assume that the capital share in national income is 1/3, while the labor share is 2/3; this configuration is consistent with the Cobb–Douglas production function with constant returns to scale (where factor shares sum to one).

The aggregate production function (1) in its general form does not guarantee balanced growth; achieving such a property requires imposing restrictions on the nature of technological progress. Suppose that technological change can separately affect

capital productivity $A_t^{(K)}$, labor productivity $A_t^{(L)}$, and total factor productivity $A_t^{(T)}$. Then the production function can conveniently be written as follows:

$$\tilde{F}(K_t, L_t, A_t) = A_t^{(T)} F[A_t^{(K)} K_t, A_t^{(L)} L_t], \quad (6)$$

where $A_t = (A_t^{(T)}, A_t^{(K)}, A_t^{(L)})$ is the technology vector; $A_t^{(T)}$ is “Hicks-neutral” progress, i.e., a multiplicative factor that scales the entire production function uniformly; $A_t^{(K)}$ is Solow-neutral” (or capital-augmenting) technological progress; and $A_t^{(L)}$ is “Harrod-neutral” (or labor-augmenting) technological progress [17].

It should be noted that in neoclassical models with constant returns to scale and competitive markets, the conditions for balanced growth typically require Harrod-neutral (labor-augmenting) technological progress. Uzawa’s theorem [20] states that a long-run balanced growth path is one along which aggregate output, capital, and consumption grow at constant rates; this is possible only under Harrod-neutral technical progress. In other words, the production function takes the form:

$$Y_t = F(K_t, A_t L_t) = A_t L_t F\left(\frac{K_t}{A_t L_t}, 1\right) = A_t L_t f(k_t), \quad (7)$$

where $k_t \equiv \frac{K_t}{A_t L_t}$ is capital intensity per unit of effective labor. Accordingly, output per unit of effective labor is $y_t \equiv \frac{Y_t}{A_t L_t} = f(k_t)$, and the product $A_t L_t$ is interpreted as “labor in efficiency units” (i.e., the number of effective workers).

Applying a production function of form (7) to describe the evolution of capital, we obtain:

$$\begin{aligned} \dot{K}_t &= Y_t - C_t - \delta K_t = s Y_t - \delta K_t = \\ &= s F(K_t, A_t L_t) - \delta K_t. \end{aligned} \quad (8)$$

By expressing variables per effective unit of labor terms $k_t \equiv \frac{K_t}{A_t L_t}$ and using $\frac{\dot{L}_t}{L_t} = n$, $\frac{\dot{A}_t}{A_t} = g$, we obtain as follows:

$$\dot{k}_t = s f(k_t) - (n + \delta + g) k_t,$$

$$\text{where } y_t \equiv \frac{Y_t}{A_t L_t} = f(k_t). \quad (9)$$

The intuition behind “effective units” of labor is as follows: technological progress that raises labor productivity at rate g is equivalent to an increase in the number of workers by the same magnitude. Therefore, “effective labor” $A_t L_t$ grows at the combined rate $n + g$, which is reflected in equation (9) through the subtraction of the term $(n + g + \delta)k_t$. Working with variables expressed per effective worker simplifies the analysis and allows determining the steady-state level k^* from the condition $s f(k^*) = (n + g + \delta)k^*$, and subsequently recovering the levels K_t and Y_t .

EMPIRICAL TESTING OF THE MODEL IN DISCRETE AND CONTINUOUS TIME

It is important to keep in mind the model’s applicability to real-world data, which is associated with a number of challenges, including data quality and cross-country and intertemporal comparability; structural breaks and parameter instability; identification of factor effects; sensitivity of results to underlying assumptions; and the accuracy of forecasts as well as the correct interpretation of empirical findings. The Solow model is well suited to capturing long-run trends (under slowly evolving structural parameters), but it is not designed to explain short-run fluctuations, where stochastic shocks dominate.

When working with such models, the issue of data reliability and its consistency with the analytical expressions used in modelling inevitably arises. In contemporary empirical research, the Penn World Table (PWT) [21] is widely used for cross-country comparisons. PWT provides a transparent and well-documented methodology; its current official version 10.01, employed in this study, covers 183 countries over the period 1950–2019, making it particularly suitable for the analysis of long time series.

In the following section, we test empirically, using data for selected OECD economies, the extent to which the Solow model with technological progress is suitable for constructing growth trajectories and estimating steady states (see Fig. 1). Two approaches are employed: (a) a determinis-

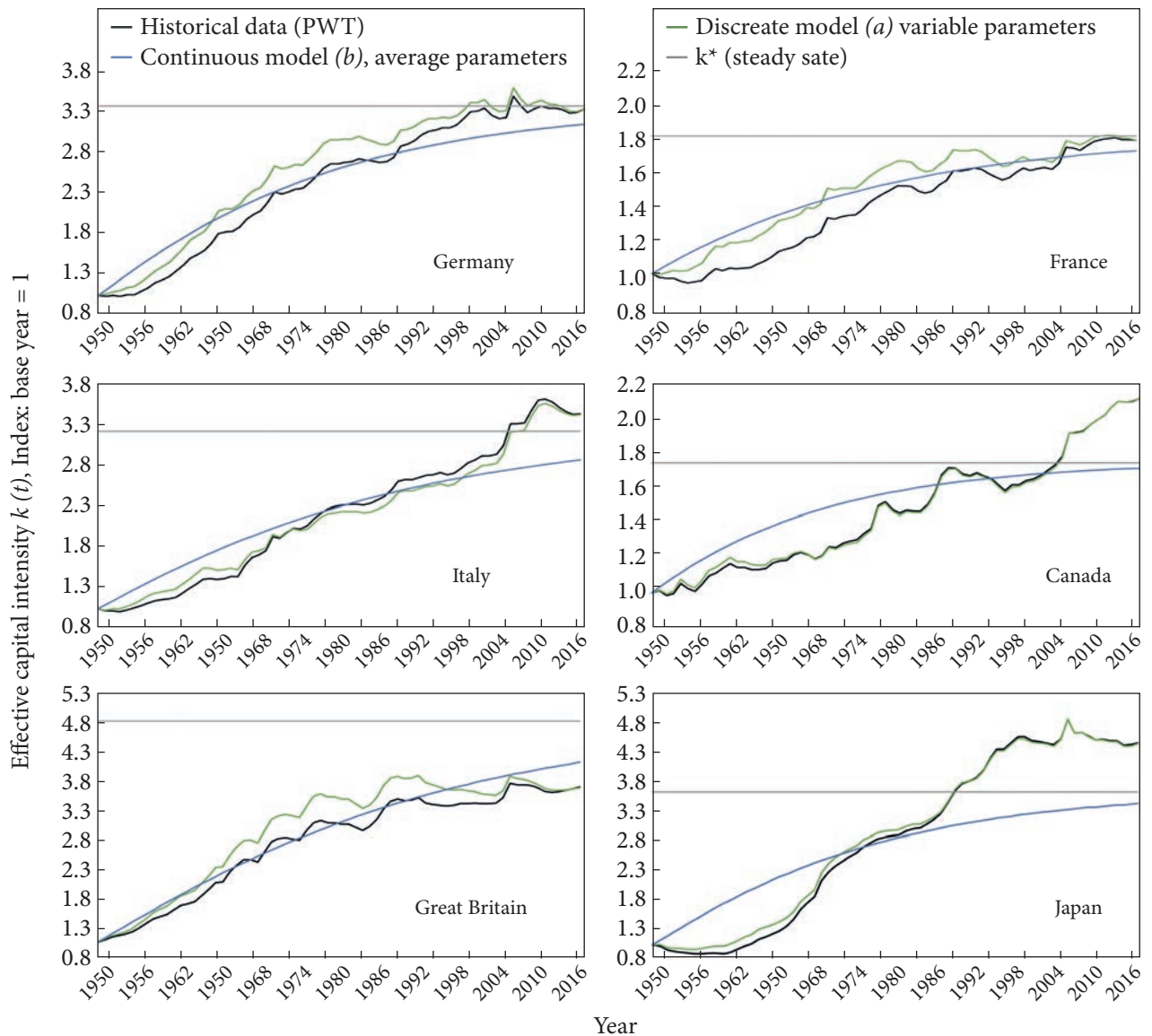


Fig. 1. Capital intensity in efficiency units in selected OECD countries, 1950–2019: historical data and model-generated trajectories

Source: Penn World Table (PWT) 10.01; authors' calculations.

tic discrete-time Solow model with time-varying structural parameters observed in each period; and (b) a continuous-time solution of the Solow model using historical average values of the structural parameters. In both cases, we use an aggregate production function with Harrod-neutral technological progress, i.e., we operate in efficiency units of labor.

To construct trajectory (a), an exact solution for capital intensity in efficiency units is required for each time period, starting from a given initial value k_0 . The key term here is “exact”: one of the advantages of the Solow model is that it admits an exact discrete-time law of motion for each time step.

Since the structural parameters s_p , n_p , δ_p , g_p , α_t vary over time, we are not working with a period-by-pe-

riod forecast k_t relative to the initial value k_0 , based on the observed parameter values at each time t rather than with the canonical stationary specification. The discrete-time law of motion for capital differs slightly from continuous-time equation (9); the reason for this distinction is shown below.

Let us begin with the budget identity:

$$\begin{aligned} Y_t &= F(K_t, A_t L_t) = C_t + I_t = \\ &= C_t + [K_{t+1} - (1 - \delta_t) K_t]. \end{aligned}$$

Dividing both sides by $A_t L_t$,

$$\frac{C_t}{A_t L_t} + \left[\frac{K_{t+1}}{A_{t+1} L_{t+1}} \cdot \frac{A_{t+1} L_{t+1}}{A_t L_t} - (1 - \delta_t) \frac{K_t}{A_t L_t} \right] = \frac{Y_t}{A_t L_t},$$

we obtain as follows:

$$c_t + [(1 + n_t)(1 + g_t) k_{t+1} - (1 - \delta_t) k_t] = f(k_t). \quad (10)$$

Since $c_t = (1 - s_t) y_t = (1 - s_t) f(k_t)$, equation (10) provides the exact discrete-time law of motion:

$$k_{t+1} = \frac{sf(k_t) + (1 - \delta_t) k_t}{(1 + n_t) \cdot (1 + g_t)}. \quad (11)$$

Using a Cobb–Douglas technology in Harrod-neutral form, when $f(k_t) = k_t^\alpha$, we arrive at the following:

$$k_{t+1} = \frac{sk_t^\alpha + (1 - \delta_t) k_t}{(1 + n_t) \cdot (1 + g_t)}. \quad (12)$$

From the Penn World Table, we extract the required empirical data and assign their corresponding roles within the model:

- ◆ *rgdpna* is real GDP in constant 2017 national prices (million 2017 USD) $\rightarrow Y_t$;
- ◆ *rnna* is capital in constant 2017 national prices (million 2017 USD) $\rightarrow K_t$;
- ◆ *emp* is number of employed persons (million);
- ◆ *avh* are average annual hours worked per employed person (hours per year);
- ◆ *hc* is human capital index (based on years of learning and returns to education);
- ◆ *labsh* is labor income share in GDP at current national prices $\rightarrow 1 - \alpha_t$ (thus, $\alpha_t = 1 - \text{labsh}_t$);
- ◆ *delta* is average depreciation rate of fixed capital $\rightarrow \delta_t$;

- ◆ *rdana* is real domestic absorption at constant 2017 national prices, i.e. $C_t + I_t + G_t$; in the standard Solow model this aggregate is not a core variable, but it is useful for reconciling/checking accounting balances;
- ◆ *rconna* is real final consumption expenditure at constant 2017 national prices (million USD, 2017) $\rightarrow C_t$.

NOTES ON THE MEASUREMENT OF LABOR

In the canonical specification, employment *emp* is typically used to represent the number of workers. However, we aim to ensure that the Solow residual (TFP) more closely reflects purely technological factors and does not absorb variations in human capital.

Therefore, in this model, “labor” is defined as efficiency-adjusted labor input in hours, weighted by labor quality: $L_t = (\text{emp}_t) \cdot (\text{avh}_t) \cdot (\text{hc}_t)$. That is, $A_t L_t$ represents “effective labor” measured in hours multiplied by a quality index.

Accordingly, part of the model’s structural parameters (δ_t, α_t) is taken directly from the PWT, while the remaining variables are computed for each period t as follows:

- ◆ labor growth rate $n_t = (L_t - L_{t-1}) / L_{t-1}$;
 - ◆ savings rate $s_t = I_t / Y_t$, $I_t = K_t - (1 - \delta_t) K_{t-1}$,
accordingly, $s_t = \frac{rnna_t - (1 - \delta_t) \cdot rnna_{t-1}}{rgdpna_t}$;
 - ◆ technology growth rate $g_t = \Delta \ln A_t$, which is best computed in log-differences using Törnqvist weights $g_t \approx \Delta \ln Y_t - \bar{\alpha}_t \Delta \ln K_t - (1 - \bar{\alpha}_t) \Delta \ln L_t$, where $\bar{\alpha}_t = (\alpha_t + \alpha_{t-1}) / 2$. Of course, one can alternatively use the pre-computed PWT series *rtfpna* (TFP at constant national prices, 2017 = 1). However, we choose to construct directly g_t in order to ensure consistency with our definition of labor $L_t = (\text{emp}_t) \cdot (\text{avh}_t) \cdot (\text{hc}_t)$. This approach also allows a direct decomposition of growth into factor contributions and TFP for each period.
- We are less interested in the absolute levels K_t and L_t ; the key object is the growth rate of capital

intensity $k_t = K_t/(A_t L_t)$ in efficiency units. For visualization purposes, it is convenient to index the series (setting the base year equal to 1), while performing all computations in levels in order to correctly compute k_t and avoid distortions.

To make the model and data comparable, we normalize productivity by selecting a base year t_0 and setting $A_{t_0} = 1$ after which the trajectory A_t is reconstructed from growth rates g_t . In efficiency-labor terms, the Cobb–Douglas production function takes the form $y_t = f(k_t) = \phi k_t^{\alpha}$, where $\phi > 0$ is a constant scale parameter (by default $\phi = 1$, later calibrated).

To construct trajectory (b) (the Solow model in continuous time with historical average structural parameters), we use the analytical solution of the ordinary differential equation. A closed-form solution is available specifically under the Cobb–Douglas specification, since the equation transforms into a Bernoulli equation and can be linearized via a change of variables [22]. Equation (9) then takes the form:

$$\dot{k}_t = s\phi k_t^{\alpha} - (n + \delta + g)k_t. \quad (13)$$

Let us introduce variable $z_t = k_t^{1-\alpha}$. In this case, $\dot{z}_t = (1-\alpha)k_t^{-\alpha} \dot{k}_t$. Multiplying (13) by $(1-\alpha)k_t^{-\alpha}$, we obtain the following linear differential equation:

$$\dot{z}_t = (1-\alpha)s\phi - (1-\alpha)(n + \delta + g)z_t.$$

From this, the general solution $z_t = Me^{-(1-\alpha)(n+\delta+g)t} + \frac{s\phi}{n+\delta+g}$ is obtained, where the constant M is determined by the initial condition $k(0) = k_0$:

$$k_0^{1-\alpha} = M + \frac{s\phi}{n+\delta+g} \Rightarrow M = k_0^{1-\alpha} - \frac{s\phi}{n+\delta+g}.$$

Returning to k_t , we obtain as follows:

$$k_t^{1-\alpha} = \left(k_0^{1-\alpha} - \frac{s\phi}{n+\delta+g} \right) e^{-(1-\alpha)(n+\delta+g)t} + \frac{s\phi}{n+\delta+g},$$

or, raising both sides to the power $1/(1-\alpha)$, arrive at:

$$k_t = \left[\left(k_0^{1-\alpha} - \frac{s\phi}{n+\delta+g} \right) e^{-(1-\alpha)(n+\delta+g)t} + \frac{s\phi}{n+\delta+g} \right]^{\frac{1}{1-\alpha}}. \quad (14)$$

Thus, to construct trajectory (b), it is sufficient to specify the initial value k_0 and fix constant parameters s , n , δ , g , and α . These are taken as historical averages over the chosen period: arithmetic means for shares and rates (s , δ , α), and geometric means for growth rates (n , g). The trajectory is then determined by the analytical solution (14).

To assess how far the actual and model trajectories (cases (a) and (b)) are from the steady state k^* , we define k^* as the fixed point of the discrete law of motion (11) under constant parameters s , n , δ , g , α :

$$k^* = \frac{sf(k^*) + ((1-\delta)k^*)}{(1+n)(1+g)} \Rightarrow \\ \Rightarrow [(1+n)(1+g) - (1-\delta)]k^* = sf(k^*).$$

From this, we obtain as follows:

$$[n + \delta + (1+n)g]k^* = sf(k^*). \quad (15)$$

For the Cobb–Douglas specification in efficiency units $y = f(k) = \phi k^{\alpha}$, we have as follows:

$$k^* = \left(\frac{s\phi}{n + \delta + (1+n)g} \right)^{\frac{1}{1-\alpha}}. \quad (16)$$

Thus, by applying the methodology described above to compute the steady state k^* and to construct the evolution of k_t using two approaches — discrete and continuous — we obtain the results presented in Fig. 1. This allows us to immediately formulate several theoretical and methodological conclusions regarding the consistency between historical data and the simulated dynamics.

First, the discrete-time trajectory (a) with time-varying structural parameters s_t , n_t , δ_t , g_t , α_t reproduces the observed dynamics more accurately than the continuous-time specification (b), which relies on historical average values of these parameters. This is

expected, since parameter averaging smooths periods of acceleration and deceleration and, given the nonlinearity of the model $f(k) = \phi k^\alpha$, systematically shifts the trajectory away from that obtained under annual parameter updates. In contrast, the discrete model explicitly incorporates yearly changes in s_t , n_t , δ_t , g_t , α_t at each period, and therefore captures both local shocks and structural shifts more precisely.

Second, it should be emphasized that the steady-state level k^* computed on the basis of average parameters should be interpreted only as an approximate benchmark. Due to economic reforms, demographic transitions, investment cycles, innovation, and technological progress, structural parameters change substantially over time; accordingly, the steady-state equilibrium is itself time-varying and shifts upward in practice (as illustrated, in particular, by the graphs for Italy and Canada). It is worth noting that within the Solow framework, sustained balanced growth is driven by Harrod-neutral technological progress: it is precisely this mechanism that explains why economies do not converge to a fixed level over long horizons but continue to grow.

To make the steady-state benchmark more informative, it is advisable to divide long horizons (e.g., 70 years) into shorter subperiods (e.g., 10-year intervals) and compute a rolling steady state based on average parameter values within each period.

Third, if the scale parameter ϕ is calibrated such that, in the final period, the discrete trajectory k_t coincides with the observed level, we obtain a typical pattern of convergence toward the steady state: over the long run, k_t increases, its growth rate gradually declines, and the discrete-time trajectory closely tracks the data.

Fourth, the long-run growth rate of output per worker is determined primarily by the rate of technological progress g . As the economy approaches the steady state in terms of capital per effective worker, additional capital accumulation contributes progressively less to growth. For mature economies, increasing s_t does not generate productivity gains comparable to those driven by higher g . This leads to a focus on R&D, innovation, and technology diffusion rather than on “mechanical” increases

in investment. In addition, the discrete-time model captures innovation waves quite well, as reflected in the convergence between observed and model-generated trajectories (or at least in the similarity of their direction and magnitude of changes, even when perfect overlap of the curves is not achieved).

The Cobb–Douglas specification of the Solow model with Harrod-neutral technology typically generates reasonable trajectories and smooth convergence for mature economies. However, for transition economies characterized by frequent structural breaks, this specification may fail to adequately capture observed dynamics. For instance, applying the Solow model to Ukraine using PWT data for 1990–2019 yields an estimated steady-state level that significantly exceeds the actual observed trajectory. This implies that, given average structural parameters, the economy would be expected to grow faster than it does in practice.

The sources of this discrepancy are evident: the complex and uneven process of institutional development, the loss of territories and resources due to military conflict, and the outflow of human capital through migration, among others. The Solow model is not designed to capture such factors: it is a deterministic framework intended to describe long-run trend regularities and does not incorporate mechanisms for large non-trend shocks. It also lacks explicit parameters capturing institutional change, which can only enter indirectly through the Solow residual (TFP). This leads to an important practical implication: for “complex” economies, the steady state should be treated as a time-varying k_t^* that evolves with changes in structural parameters rather than a fixed value k^* . Alternatively, long time horizons can be divided into subperiods with separate average parameter values in order to better align the model with empirical dynamics.

Despite the fact that the Solow model explains long-run growth through technological progress, this progress is exogenous and lacks microeconomic foundations. The assumptions of perfect competition and constant returns to scale imply the absence of rents above normal profits, and therefore no mechanism exists within the model to ge-

nerate private incentives for R&D. As a result, exogenous technological progress in the neoclassical framework is not equivalent to Schumpeterian innovation, which is understood as deliberate innovation undertaken by agents to obtain economic profit. Capturing such incentives requires endogenizing knowledge and relaxing the assumption of perfect competition, allowing instead for monopolistic competition and temporary innovation rents (profits from the introduction of new technologies). Within the neoclassical Solow framework, this issue remains fundamentally unresolved.

GROWTH FACTOR DECOMPOSITION AND TFP ESTIMATION

By analyzing the production function in the Solow model, it can be viewed from a different perspective — as a tool for decomposing output growth into contributions from factor inputs, where two of them are capital and labor, and the third is an unobserved residual, commonly interpreted as knowledge, technological progress, or innovation. In his empirical work on U.S. data, R. Solow showed that over the period 1909–1949, approximately 87.5% of the growth in output per man-hour was attributable to this residual (later known as the “Solow residual”), while 12.5% was due to capital deepening [23]. The Solow residual is also referred to as TFP, since it governs the efficiency with which labor and capital are used. At the same time, TFP encompasses not only “pure” technology but also other hard-to-measure factors.

Theoretically, TFP represents the rate of shift of the production function over time due to technological and organizational improvements. In the standard value-added framework, it is measured as output growth minus the weighted growth rates of factor inputs. In discrete time, it is convenient to write:

$$\ln(A_{t+1} / A_t) \approx \ln(Y_{t+1} / Y_t) - \bar{\alpha}_t \ln(K_{t+1} / K_t) - (1 - \bar{\alpha}_t) \ln(L_{t+1} / L_t), \quad (17)$$

where $\bar{\alpha}_t \equiv (\alpha_t + \alpha_{t+1}) / 2$ are Törnqvist weights (average value-added shares of capital over the period $t \rightarrow t + 1$).

The choice of methodology determines what is included in TFP. Equation (17) represents the classical formulation (growth accounting in value-added terms) in the spirit of R. Solow [23] or E. Denison [24], which relies on weighted growth rates of factor inputs and typically leaves a substantial “Solow residual.” Accordingly, a significant portion of long-run aggregate output growth was attributed to technological progress.

At the same time, alternative approaches exist. In particular, a notable example is the index-number approach developed by D. Jorgenson and Z. Griliches within the framework of national account-

ing. The starting point is the accounting identity of values $\sum_{i=1}^m q_i Y_i = \sum_{j=1}^n p_j X_j$, where the left-hand side represents the value of gross output (prices q_i and quantities Y_i), while the right-hand side represents the value of all utilized inputs (prices p_j and quantities X_j) [25]. For this reason, under the assumptions of perfect competition and constant returns to scale, TFP is measured as the difference between output growth and the weighted sum of the growth rates of all inputs. This framework directly links industry-level data with macroeconomic indicators and allows a more detailed decomposition of growth contributions, including labor adjusted for quality, services of different types of capital, intermediate inputs, and other production factors. The principal advantage of this approach is that it “purifies” TFP: factors that would otherwise be absorbed into the Solow residual under more aggregated methodologies are instead attributed to measurable inputs. As a result, the estimated TFP is typically smaller in magnitude but methodologically closer to what can reasonably be interpreted as a pure technological or organizational shift.

In the present study, we deliberately employ the “canonical” value-added growth decomposition in the tradition of R. Solow [23]. A highly detailed decomposition of growth contributions — including industry accounts, intermediate inputs, and related factors — is beyond the scope of this pa-

per. Nevertheless, we regard the methodology developed by D. Jorgenson and Z. Griliches as highly valuable and reserve its application for future research. Remaining within the Solow framework, we rely on the Penn World Table (PWT) database to estimate TFP. Aggregate output is measured by the variable *rgdpna*, introduced earlier, while the production factors are represented by the following indicators:

- ◆ *rkna* are capital services at constant 2017 national prices (2017 = 1). In this regard, we follow the recommendation of R. Barro and X. Sala-i-Martin that, ideally, the contribution of capital should be measured by the flow of services generated by physical capital rather than by the capital stock itself [19];
- ◆ *emp · avh · hc* like before, we assume that labor input is measured as the number of employed persons multiplied by the average annual hours worked and adjusted for worker quality through the human capital index, thereby capturing both the quantity and quality dimensions of labor input.

The results of the growth accounting exercise for the selected countries are presented in Table 1. Column (1) reports the average value of the capital share, α . Column (3) presents the average annual growth rate of real GDP. Column (4) shows the contribution of labor, calculated as the product of labor's income share ($1 - \alpha$) and the growth rate of employment. Column (5) reports the contribution of capital, calculated as the product of the capital share α and the growth rate of capital services. Co-

lumn (6) presents the growth rate of TFP. The figures in parentheses indicate the proportion of total GDP growth explained by the respective factor.

Our calculations have shown that using capital services (*rkna*) instead of the capital stock (*rnna*) reduces the estimated share of TFP by 4–8 percentage points, while increasing the contribution of capital by a corresponding amount. This finding suggests that: (a) TFP is sensitive to measurement methodology and should not automatically be interpreted as “pure technology”; (b) capital services more accurately capture the quality and composition of assets and therefore provide a better approximation of capital's actual contribution to growth; and (c) cross-country and intertemporal comparisons should be conducted only on the basis of harmonized input definitions, otherwise conclusions regarding the role of TFP may be distorted.

How should the results of Table 1 be interpreted? First, capital emerges as the principal driver of economic growth in all countries examined, accounting for between 37% of growth in Canada and 68% in China. The contribution of labor ranges from 7% to 40%, depending on demographic trends, changes in hours worked, and the accumulation of human capital. In mature economies, reductions in hours worked often offset modest increases in employment and labor quality, resulting in a relatively small overall contribution of labor. Under the growth accounting framework, the “Solow residual” — that is, the portion of growth not explained by labor and capital — is attributed to TFP,

Table 1. Decomposition of Contributors to Growth: Capital, Labor, and TFP (for selected countries)

Country	Years	GDP growth rate	Labor share	Capital share	TFP share
Great Britain ($\alpha = 0.43$)	1951—1970	0.0263	0.0007 (3%)	0.0229 (87%)	0.0027 (10%)
	1971—1990	0.0249	0.0045 (18%)	0.0146 (58%)	0.0059 (24%)
	1991—2019	0.0198	0.0066 (33%)	0.0106 (54%)	0.0026 (13%)
	1951—2019	0.0232	0.0043 (19%)	0.0153 (66%)	0.0036 (15%)

Country	Years	GDP growth rate	Labor share	Capital share	TFP share
Canada ($\alpha = 0.29$)	1951—1970	0.0471	0.0108 (23%)	0.0120 (25%)	0.0243 (52%)
	1971—1990	0.0325	0.0201 (62%)	0.0135 (41%)	−0.0010 (−3%)
	1991—2019	0.0226	0.0098 (44%)	0.0109 (48%)	0.0019 (8%)
	1951—2019	0.0326	0.0131 (40%)	0.0120 (37%)	0.0075 (23%)
China ($\alpha = 0.41$)	1951—1970	0.0372	0.0230 (62%)	0.0259 (69%)	−0.0117 (−31%)
	1971—1990	0.0554	0.0257 (46%)	0.0336 (61%)	−0.0039 (−7%)
	1991—2019	0.0650	0.0115 (18%)	0.0472 (72%)	0.0063 (10%)
	1951—2019	0.0539	0.0192 (36%)	0.0368 (68%)	−0.0021 (−4%)
Germany ($\alpha = 0.35$)	1951—1970	0.0612	0.0062 (10%)	0.0242 (40%)	0.0307 (50%)
	1971—1990	0.0258	−0.0007 (−3%)	0.0132 (51%)	0.0132 (52%)
	1991—2019	0.0147	0.0012 (8%)	0.0075 (51%)	0.0060 (41%)
	1951—2019	0.0314	0.0021 (7%)	0.0140 (45%)	0.0153 (48%)
USA ($\alpha = 0.38$)	1951—1970	0.0386	0.0131 (34%)	0.0140 (36%)	0.0114 (30%)
	1971—1990	0.0319	0.0135 (42%)	0.0136 (43%)	0.0048 (15%)
	1991—2019	0.0246	0.0067 (28%)	0.0113 (46%)	0.0065 (26%)
	1951—2019	0.0307	0.0106 (34%)	0.0127 (42%)	0.0074 (24%)
Japan ($\alpha = 0.40$)	1951—1970	0.0885	0.0182 (20%)	0.0351 (40%)	0.0352 (40%)
	1971—1990	0.0440	0.0074 (17%)	0.0283 (64%)	0.0083 (19%)
	1991—2019	0.0096	0.0002 (2%)	0.0089 (93%)	0.0005 (5%)
	1951—2019	0.0424	0.0075 (18%)	0.0221 (52%)	0.0128 (30%)

which is commonly interpreted as reflecting the effects of technological and organizational improvements. According to our estimates for the period 1951–2019, the contribution of TFP remains substantial, accounting for approximately 15% of growth in the United Kingdom, 23% in Canada, 24% in the United States, 30% in Japan, and 48% in Germany. China constitutes an exception, with an average TFP contribution of approximately –4%, largely due to negative TFP growth during the decades preceding the economic reforms.

Secondly, these results confirm a slowdown in growth rates in mature economies — precisely what the neoclassical model predicts. In the immediate postwar decades, growth rates were high; in particular, in Germany and Japan a substantial share of output growth was accounted for by TFP (50% and 40% respectively), which is consistent with institutional development and rapid technology diffusion. In China during this period, TFP is negative, reflecting institutional constraints and inefficiencies of the planned economy. In 1971–1990, following the oil shocks, a general decline in productivity occurred: the role of capital increased, while the contribution of TFP gradually decreased. The last two decades show significant cross-country divergence and require more careful analysis. In the context of automation, the rising role of knowledge, and digitalization, “innovation gains” are partially recorded as capital services or labor quality improvements; as a result, measured TFP is redistributed across components and becomes a less unambiguous indicator of “pure technology.” This calls for cautious interpretation and harmonized measurement methodologies.

A high TFP share should be interpreted as an indirect indicator of innovation and of improved efficiency in the use of labor and capital, rather than as a simple “better/worse” metric. At the same time, it should be interpreted alongside the absolute growth rate: even when its share is large, the absolute contribution of TFP may remain modest. It is also important to compare this indicator with the level of investment: sustained growth requires sufficient capital accumulation, and an excessive-

ly large role of TFP in the context of weak investment may indicate the “consumption” of future productive potential. Finally, measurement quality must be taken into account in order to avoid overstated “residuals” that do not, in fact, reflect technological progress.

An attempt to equate the Solow residual with a factor of technological change, despite receiving support from representatives of the neoclassical school, has at the same time faced sharp criticism from proponents of heterodox approaches. According to G. Roeggiers, representing technological progress as an indeterminate residual of an aggregated production function is a convenient methodological fiction: “De facto, such reductionism transforms technical progress, one of the key driving forces of history, into nothing more than an obscure component of an equation intended to describe economic change” [26].

Well-argued criticism of TFP as a direct measure of innovation is present in the work of many authors. In particular, study [27] shows that traditional growth accounting relies on unrealistic assumptions — perfect competition and the temporal coincidence of costs and revenues (wages and material inputs are assumed to accrue in the same period as output). As a result, the method underestimates intermediate inputs (and in some cases labor), overestimates capital, and the resulting TFP estimate may be biased in either direction.

In [28], the authors emphasize that TFP is not synonymous with technological progress, and that its estimates are highly sensitive to methodology. Therefore, alongside conventional growth accounting, more advanced methods and robustness checks should be applied to disentangle efficiency effects from technology itself. In [29], it is stated that TFP as the Solow residual is rather a “measure of our ignorance” than pure technical progress; conclusions depend on assumptions and measurement choices, while the complementarity of factors is often ignored. In [30], the authors demonstrate that production function estimates and TFP are largely transformations of the accounting identity of the system of national ac-

counts rather than tests of a “law of production.” Hence, the Solow residual reflects the algebra of income decomposition rather than “pure” technical progress. This underpins a broader critique of a TFP-centered paradigm as one that generates an “illusion of knowledge.”

The transition to a post-industrial society requires a broader definition of capital that includes intangible assets. This allows part of the “unexplained” growth to be reallocated from the Solow residual into measurable factors, thereby reducing the magnitude of TFP. However, accounting for the innovation component in growth remains a complex methodological challenge. On the one hand, part of private-sector spending on technological development is already capitalized as intangible capital; on the other hand, a significant share of expenditures is still treated as current costs (for example, employee training, certain organizational changes, and some components of public R&D), and therefore implicitly enters TFP.

A systematic attempt to address this issue was proposed by C. Corrado and co-authors. They expanded the concept of capital by distinguishing three categories of intangible assets: computerized information (software, databases); innovative property (R&D, mineral exploration, copyrights and licenses, and other product development costs); and economic competencies (brand equity, firm-specific human capital, and organizational capital). According to their estimates for the United States (mid-1990s to 2000s), the inclusion of these assets significantly reduces the share of TFP in productivity growth (approximately from two-thirds to one-third) [31]. To reduce the tension between models and empirical data, the contemporary literature moves in this direction by improving the measurement of production factors (including employment adjusted for human capital, capital services, and capacity utilization adjustments), incorporating a broader set of inputs and intermediate costs rather than relying solely on value added, and capitalizing intangible assets.

Based on the conducted research, the following main conclusions are formulated.

1. Under standard assumptions of the neoclassical model with constant returns to scale and competitive markets, a long-run balanced growth path is sustained by Harrod-neutral technical progress. In empirical implementation, this implies working in efficiency units of labor using normalized variables and growth rates. After estimation, results are then transformed back into levels for interpretation.

2. The discrete version of the model with time-varying parameters s_t , n_t , δ_t , g_t , α_t has been found to better replicate historical data. The continuous-time specification with averaged parameters adequately captures the general trend and smooths periods of acceleration and deceleration; therefore, it primarily serves to illustrate convergence toward the steady state determined by structural parameters.

3. The steady state should be treated as a time-varying magnitude. A steady-state level calculated using long-run averages represents only a rough approximation. In the presence of institutional reforms, demographic shifts, technological innovations, and other macroeconomic shocks, it is more appropriate to employ moving averages or to explicitly model a time-varying level of capital intensity k_t^* .

4. **Suitability of the Solow model.** The Cobb–Douglas specification with Harrod-neutral technical progress generally performs well for mature economies. In contrast, for transition economies characterized by frequent structural breaks, it tends to produce biased steady-state estimates and provides a poorer fit to the data unless parameters are calibrated over time.

5. **Growth accounting for selected countries.** Growth accounting results for selected countries (predominantly OECD member states) indicate that capital has been the main source of economic growth. The contribution of labor has been moderate, while TFP has remained substantial in most cases. Exceptions are observed during periods of institutional distortions and large-scale structural reforms.

6. **Definition of input factors.** The definition of input variables is critically important. Substi-

tuting capital stock with capital services reduces the estimated contribution of TFP and correspondingly increases the measured contribution of capital. Similarly, alternative definitions of labor significantly affect the decomposition results in growth accounting exercises.

7. Interpretation of TFP. TFP must be interpreted with caution, as it represents a composite measure that combines the effects of technological change, organizational improvements, allocative efficiency, capacity utilization, and measurement errors. Expanding the concept of capital to include intangible assets partially “purifies” TFP by real-locating part of growth from the residual to measurable factors.

8. Limitations of the Solow approach. The Solow model does not allow for the analysis of innovation as a purposeful activity of economic agents:

the exogeneity of technology and the assumption of perfect competition make its results primarily trend-based. For questions of innovation policy, extensions are required — particularly endogenous growth models that incorporate innovation rents and introduce mechanisms of imperfect (monopolistic) competition.

Future research directions include a more detailed estimation of capital services (distinguishing between tangible and intangible capital and decomposing assets by classes) and the application of labor quality adjustment methods (including alternative human capital indices). The objective is to reduce “pseudo-TFP” and to more precisely isolate the pure technological component of growth. In the era of digitalization, the simple capital–labor dichotomy is no longer sufficient; broader approaches to measuring factor contributions are required.

REFERENCES

1. Young, A. A. (1928). Increasing Returns and Economic Progress. *The Economic Journal*, 38(152), 527—542. <https://doi.org/10.2307/2224097>
2. Harrod, R. F. (1939). An Essay in Dynamic Theory. *The Economic Journal*, 49(193), 14—33. <https://doi.org/10.2307/2225181>
3. Domar, E. D. (1946). Capital Expansion, Rate of Growth, and Employment. *Econometrica*, 14, 137—147. <https://doi.org/10.2307/1905364>
4. Solow, R. M. (1956). A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*, 70(1), 65—94. <https://doi.org/10.2307/1884513>
5. Swan, T. W. (1956). Economic Growth and Capital Accumulation. *Economic Record*, 32, 334—361. <http://dx.doi.org/10.1111/j.1475-4932.1956.tb00434.x>
6. de la Fontejne, M. R. (2018). *Why the concept of Hicks, Harrod, Solow neutral and even non-neutral augmented technical progress is flawed in principle in any economic model* (MPRA Paper No. 107730). University Library of Munich, Germany. URL: <https://mpra.ub.uni-muenchen.de/107730/> (Last accessed: 24.08.2025).
7. Mert, M. (2021). Economic growth under Solow-neutrality. *Economic Research-Ekonomska Istraživanja*, 34(1), 3440—3467. <https://doi.org/10.1080/1331677X.2021.1875860>
8. Cabello, M. (2024, November 12). *A note on bounded growth with CES functions*. SSRN. <https://doi.org/10.2139/ssrn.5018671>
9. Senouci, M. (2024). *The Solow model under localized technical change* (Working Paper No. halshs-04627529). HAL Open Archive. URL: <https://shs.hal.science/halshs-04627529v1/document> (Last accessed: 24.08.2025).
10. Jefferson, G. H. (2024, November 11). *Making growth theory relevant: Connecting capital — labor augmentation, theory and empirical research, and economics and nature*. <https://doi.org/10.2139/ssrn.5075634>
11. Kim, Y. E., Loayza, N. V. (2019). *Productivity Growth: Patterns and Determinants across the World*. World Bank, Washington, DC.
12. Harb, G., Bassil, C. (2023). TFP in the Manufacturing Sector: Long-Term Dynamics, Country and Regional Comparative Analysis. *Economies*, 11(2), 34. <https://doi.org/10.3390/economies11020034>

13. Silveira, N. J. C., Ferraz, D., Mello, D. S., Polloni-Silva, E., Moralles, H. F., Rebelatto, D. A. do N. (2023). Calculating models for total factor productivity measurement. *Exacta*, 21(2), 297—315. <https://doi.org/10.5585/exactaep.2021.18140>
14. Wooldridge, J. M. (2009). On estimating firm-level production functions using proxy variables to control for unobservables. *Economics Letters*, 104, 112—114. <https://doi.org/10.1016/j.econlet.2009.04.026>
15. Sequeira, Tiago, Morão, Hugo. (2020). Growth accounting and regressions: New approach and results. *International Economics*, 162, 67—79. <https://doi.org/10.1016/j.inteco.2020.01.003>
16. Barral, P., Senouci, M. (2018, August). Walking on two legs: Growth accounting with labor-saving and capital-saving technical change. *Paper presented at the 71st European Meeting of the Econometric Society (ESEM) (August 27—31, 2018, Cologne, Germany)*. HAL Id: hal-01709599.
17. Acemoglu, D. (2009). *Introduction to modern economic growth*. Princeton, NJ [u.a.]: Princeton Univ. Press.
18. Kaldor, N. (1963). *Capital Accumulation and Economic Growth*. In: *Proceedings of a Conference Held by the International Economics Association* (Eds. F.A. Lutz, D.C. Hague). MacMillan, London, 177—222.
19. Barro, R. J., Sala-i-Martin, X. (2004). *Economic growth*. 2nd Edition, Vol. 1. Cambridge.
20. Uzawa, H. (1961). Neutral inventions and the stability of growth equilibrium. *The Review of Economic Studies*, 28(2), 117—124. <https://doi.org/10.2307/2295709>
21. Feenstra, R. C., Inklaar, R., Timmer, M. P. (2023). *Penn World Table* (Version 10.01). Groningen Growth and Development Centre, University of Groningen. <https://doi.org/10.34894/QT5BCC>
22. Novales, A., Fernández, E., Ruiz, J. (2022, December). *Economic growth* (3rd ed., Springer Texts in Business and Economics). Springer.
23. Solow, R. M. (1957). Technical Change and the Aggregate Production Function. *The Review of Economics and Statistics*, 39(3), 312—320. <https://doi.org/10.2307/1926047>
24. Denison, E. F. (1962). *The sources of economic growth in the United States and the alternatives before us*. Committee for Economic Development.
25. Jorgenson, D. W., Griliches, Z. (1967). The explanation of productivity change. *The Review of Economic Studies*, 34(3), 249—283. URL: <https://apps.bea.gov/scb/pdf/1972/0572cont.pdf> (Last accessed: 24.08.2025).
26. Rosegger, G. (1995). Technical change without humans: Innovation in the neoclassical economic theory. In: *Revolutionary changes in understanding man and society* (Theory and Decision Library, Vol. 21). (Ed. J. Götschl). Springer. https://doi.org/10.1007/978-94-011-0369-5_3
27. Louwerse, J. (2018, February 20). The Problems with Growth Accounting: Unrealistic Assumptions and Biased Results. *Business Economics*. URL: <http://hdl.handle.net/2105/41623> (Last accessed: 24.08.2025).
28. Crafts, N., Woltjer, P. (2021). Growth accounting in economic history: Findings, lessons and new directions. *Journal of Economic Surveys*, 35, 670—696. <https://doi.org/10.1111/joes.12348>
29. Callaghan, Chris W. (2022). Growth accounting, development accounting and cross-country growth regressions: A conceptual review essay. *Journal for Transdisciplinary Research in Southern Africa*, 18(1), 1—9. <https://doi.org/10.4102/td.v18i1.1051>
30. Felipe, J., McCombie, J. S. L. (2019). *The illusions of calculating total factor productivity and testing growth models: From Cobb–Douglas to Solow and Romer* (ADB Economics Working Paper Series No. 596). Asian Development Bank. <https://doi.org/10.2139/ssrn.3377758>
31. Corrado, C., Hulten, C., Sichel, D. (2009). Intangible capital and U.S. economic growth. *Review of Income and Wealth*, 55, 661—685. <https://doi.org/10.1111/j.1475-4991.2009.00343.x>

Received 24.08.2025

Revised 22.01.2026

Accepted 03.02.2026

Т.О. Пасічник (<https://orcid.org/0000-0002-7284-7345>)

Університет митної справи та фінансів,
вул. Вернадського 2/4, Дніпро, 49000, Україна,
+380 56 745 5596, umsf.ua@gmail.com

ЗБАЛАНСОВАНЕ ЗРОСТАННЯ У МОДЕЛІ СОЛОУ–СВОНА: РОЛЬ ХАРРОД-НЕЙТРАЛЬНОГО ТЕХНІЧНОГО ПРОГРЕСУ

Вступ. Робочою моделлю макроаналізу є модель Солоу–Свона з неокласичною виробничою функцією із постійною віддачею від масштабу і спадною граничною продуктивністю факторів.

Проблематика. Узгодження моделі з довгими часовими рядами потребує уточнення природи технічних змін і способів вимірювання внесків факторів. Проблематика має як теоретичну, так і прикладну вагу: дискусії про «уповільнення продуктивності» та роль нематеріальних активів безпосередньо впливають на оцінку джерел зростання і, відповідно, на економічну політику.

Мета. Розробити й емпірично перевірити методологію побудови траєкторій зростання у моделі Солоу–Свона з Харрод-нейтральним технічним прогресом: а) дискретна версія з параметрами, що змінюються у часі; б) безперервна версія з усередненими параметрами, а також виконати облік зростання з кількома підходами до визначення факторів для відокремлення внеску технічного прогресу.

Матеріали й методи. Основним джерелом макроекономічних рядів є *Penn World Table* (10.01). Аналіз виконано у двох постановках (дискретній і безперервній). Темпи технічного прогресу оцінювали через «залишок Солоу», перевіряли чутливість до вибору показників праці, капіталу та факторних часток.

Результати. Специфікація Кобба–Дугласа з Харрод-нейтральністю зазвичай добре описує зрілі економіки. Показано, що для перехідних економік із частими структурними зламами без калібрування у часі можливі зміщення оцінок стаціонару та гірша відповідність даним.

Висновки. TFP слід тлумачити обережно: це збірна величина, що поєднує технологію, організаційні зміни, ефективність розподілу ресурсів, завантаження потужностей і похибки вимірювання. Розширення поняття капіталу на нематеріальні активи частково «очищує» TFP, переносючи частину зростання із залишку до вимірюваних факторів. Обмеження: модель з екзогенним технічним прогресом і досконалою конкуренцією не описує інновації як цілеспрямовану діяльність; для цього потрібні моделі ендогенного зростання, що припускають інноваційні ренти.

Ключові слова: збалансоване зростання, облік зростання, нематеріальний капітал, факторні частки, технічний прогрес, інноваційний розвиток.